

Analysis of deep inelastic scattering with z -dependent scale

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Abstract. Evolution of the parton densities at NLO in α_S using $\tilde{W}^2 = Q^2(1-z)/z$ instead of the usual Q^2 for the scale of the running coupling α_S is investigated. While this renormalisation scale change was originally proposed as the relevant one for $x \rightarrow 1$, we explore the consequences for all x with this choice. While it leads to no improvement to the description of DIS data, the nature of the gluon at low x , low Q^2 is different, avoiding the need for a ‘valence-like’ gluon.

1 Motivation – non-singlet evolution

In deep inelastic scattering (DIS) the Q^2 dependence of flavour non-singlet quantities is quite straightforward. Taking a non-singlet structure function, e.g. $F_{NS}^+ = F_2^{ep} - F_2^{en}$ or $F_{NS}^- = F_2^{\nu p} - F_2^{\nu n}$ then at next-to-leading order (NLO) we have

$$F_{NS}^\pm(x, t) = \tilde{q}_{NS}^\pm(x, t) + \left(\frac{\alpha_S}{4\pi}\right) B_q(z) \otimes \tilde{q}_{NS}^\pm(x/z, t) \quad (1)$$

where $t = \ln Q^2$, $\tilde{q}(x, t) = xq(x, t)$, the relevant combination of quark and antiquark densities (weighted by the appropriate charges squared) is denoted by q_{NS} and $B_q(z)$ is the quark coefficient function given, for example, in the $\overline{\text{MS}}$ scheme by

$$\begin{aligned} B_q(z) &= [\hat{B}_q(z)]_+ \\ &= \left[\hat{P}_{qq}^{(0)}(z) \left\{ \ln\left(\frac{1-z}{z}\right) - \frac{3}{2} \right\} + \frac{1}{2}(9+5z) \right]_+ \end{aligned} \quad (2)$$

where

$$P_{qq}^{(0)}(z) = [\hat{P}_{qq}^{(0)}(z)]_+ = \left[C_F \left(\frac{1+z^2}{1-z} \right) \right]_+ \quad (3)$$

is the well-known LO q - q splitting function.

Here $[...]_+$ denotes the standard regularised functions defined by

$$\int_0^1 dz [f(z)]_+ g(z) = \int_0^1 dz f(z) [g(z) - g(1)] \quad (4)$$

The NLO evolution of the non-singlet quark density is governed by the qq and $\bar{q}q$ splitting functions

$$\begin{aligned} \frac{d}{dt} \tilde{q}_{NS}^\pm(x, t) &= \left\{ \left[\left(\frac{\alpha_S}{2\pi}\right) \hat{P}_{qq}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{qq}^{(1)}(z) \right]_+ \right. \\ &\quad \pm \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{\bar{q}q}^{(1)}(z) + \delta(1-z) \\ &\quad \left. \times \int_0^1 dz \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{\bar{q}q}^{(1)}(z) \right\} \otimes \tilde{q}_{NS}^\pm(x/z, t) \end{aligned} \quad (5)$$

The NLO splitting functions take the form

$$\begin{aligned} \hat{P}_{qq}^{(1)}(z) &= C_F^2 P_F(z) + \frac{1}{2} C_F C_A P_G(z) + \frac{1}{2} C_F N_F P_{N_F}(z) \\ \hat{P}_{\bar{q}q}^{(1)}(z) &= (C_F^2 - \frac{1}{2} C_F C_A) P_A(z) \end{aligned} \quad (6)$$

and the explicit expressions for $P_F(z)$, $P_G(z)$, $P_{N_F}(z)$ and $P_A(z)$ can be found in [1] for example.

By combining (1,5), the evolution of the non-singlet structure functions to $\mathcal{O}(\alpha_S^2)$ may then be expressed in the form

$$\begin{aligned} \frac{d}{dt} F_{NS}^\pm(x, t) &= \left\{ \left[\left(\frac{\alpha_S}{2\pi}\right) \hat{P}_{qq}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \{ \hat{P}_{qq}^{(1)}(z) - \frac{1}{4} \beta_0 \hat{B}_q(z) \} \right]_+ \right. \\ &\quad \pm \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{\bar{q}q}^{(1)}(z) + \delta(1-z) \int_0^1 dz \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{\bar{q}q}^{(1)}(z) \left. \right\} \\ &\quad \otimes F_{NS}^\pm(x/z, t) \end{aligned} \quad (7)$$

If, in the above α_S is a function of t only, the running coupling can be taken outside the convolution integral and we have the usual NLO in α_S evolution as implemented in standard analyses of DIS.

Next consider the case where the relevant scale depends on z as well i.e. $\alpha_S(Q^2) \rightarrow \alpha_S(\phi(z)Q^2)$ and, in particular, the choice $\phi(z) = (1-z)/z$. The quantity $\tilde{W}^2 = Q^2(1-z)/z$ is just the com energy squared of the virtual photon-parton scattering which controls the maximum transverse momentum occurring in the ladder graphs which are summed to give the leading log contribution. As $z \rightarrow 1$, \tilde{W}^2 as well as Q^2 is large and it has been argued [3,4] that, in this region, the relevant variable to account for the large logs which arise beyond the control of the renormalisation group is W^2 or a quantity closely related.

Expanding $\alpha_S(t + \ln[\phi(z)])$ to $\mathcal{O}(\alpha_S^2)$ we get

$$\frac{\alpha_S(t + \ln \phi(z))}{2\pi} = \frac{\alpha_S(t)}{2\pi} - \frac{1}{2} \beta_0 \ln[\phi(z)] \left(\frac{\alpha_S(t)}{2\pi} \right)^2 \quad (8)$$

which means that, to this order, the change of scale of α_S is equivalent to adding an extra term inside the $[\dots]_+$ of (5) equal to

$$-\frac{1}{2}\beta_0 \left(\frac{\alpha_S(t)}{2\pi}\right)^2 \hat{P}_{qq}^{(0)}(z) \ln[\phi(z)] \quad (9)$$

The above expressions for the evolution for \tilde{q}_{NS}^\pm and F_{NS}^\pm are now to be understood with $\alpha_S \equiv \alpha_S(t, z)$ but we can see from (7,9) that to $\mathcal{O}(\alpha_S^2(t))$ the shift in scale is equivalent to a term in $\hat{B}_q(z)$ equal to $2\hat{P}_{qq}^{(0)}(z) \ln[\phi(z)]$ or a term in $\hat{P}_{qq}^{(1)}(z)$ equal to $-\frac{1}{2}\beta_0 \hat{P}_{qq}^{(0)}(z) \ln[\phi(z)]$. Thus taking $\phi(z) = (1-z)/z$ the first term in (2) is generated by the change of scale $Q^2 \rightarrow Q^2(1-z)/z$ and then to avoid double counting at $\mathcal{O}(\alpha_S^2(t))$ we should use the simpler form

$$\hat{B}_q(z) \longrightarrow -\frac{3}{2}\hat{P}_{qq}^{(0)}(z) + \frac{1}{2}(9+5z) \quad (10)$$

Since $\beta_0 = \frac{1}{4}C_F(11C_A - 2N_F)$ we note that some of the N_F dependence in the combination of the NLO NS splitting function and the quark coefficient function has been absorbed by the change of scale. One can go further and compute the form $\phi(z)$ should take in order to generate the entire N_F dependence at $\mathcal{O}(\alpha_S^2(t))$. Wong [5] showed that can be achieved by taking

$$\begin{aligned} \phi(z) &\rightarrow \tilde{\phi}(z) \\ &= \left(\frac{1-z}{z^2}\right) \exp\left[(C_F/\hat{P}_{qq}^{(0)}(z))\left(\frac{1+13z}{4}\right) - \frac{29}{12}\right] \end{aligned} \quad (11)$$

This is the BLM procedure [6] where the N_F dependence in a particular process is identified as arising from the vacuum polarisation contributions to that process and these are then entirely absorbed into the running coupling thus providing a method of summing such contributions to all orders. This procedure is therefore quite attractive but it requires the choice of the z -dependent scale to vary from process to process. In that sense the evolution of the non-singlet structure function is a different process from the singlet one. It is impractical to attempt an analysis of DIS where the choice of renormalisation scale differs for the singlet and non-singlet combinations and here we shall explore the consequences for a single choice of scale, $\phi(z) = (1-z)/z$. It is clear that this simple choice accounts, at $\mathcal{O}(\alpha_S^2)$, for large terms in the $\overline{\text{MS}}$ coefficient functions occurring both in singlet and non-singlet expressions and it is worth investigating the phenomenological consequences of the potentially large logarithms which are summed by this procedure over the entire range of x .

2 Quark singlet and gluon evolution

Consider now the evolution of a singlet structure function, $F_S(x, t)$ where

$$\begin{aligned} F_S(x, t) &= \tilde{q}_S(x, t) + \left(\frac{\alpha_S}{4\pi}\right) B_g(z) \otimes \tilde{q}_S(x/z, t) \\ &+ \left(\frac{\alpha_S}{4\pi}\right) B_g(z) \otimes \tilde{g}(x/z, t) \end{aligned} \quad (12)$$

where $\tilde{q}_S(x, t) = x \sum (q(x, t) + \bar{q}(x, t))$, and $B_g(z)$ is the gluon coefficient function given in the $\overline{\text{MS}}$ scheme by

$$B_g(z) = 2\hat{P}_{Sg}^{(0)}(z) \ln\left(\frac{1-z}{z}\right) + 2N_F[8z(1-z) - 1] \quad (13)$$

where

$$\hat{P}_{Sg}^{(0)}(z) = N_F [z^2 + (1-z)^2] \quad (14)$$

is the LO q - g splitting function.

The NLO evolution of the singlet quark density is governed by the appropriate SS and Sg splitting functions

$$\begin{aligned} \frac{d}{dt} \tilde{q}_S(x, t) &= \left\{ \left[\left(\frac{\alpha_S}{2\pi}\right) \hat{P}_{qq}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{qq}^{(1)}(z) \right]_+ \right. \\ &+ \left(\frac{\alpha_S}{2\pi}\right)^2 \left\{ \hat{P}_{\bar{q}q}^{(1)}(z) + \Delta \hat{P}_{qq}^{(1)}(z) \right\} \\ &+ \delta(1-z) \int_0^1 dz \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{\bar{q}q}^{(1)}(z) \left. \right\} \otimes \tilde{q}_S(x/z, t) \\ &+ \left\{ \left(\frac{\alpha_S}{2\pi}\right) \hat{P}_{Sg}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{P}_{Sg}^{(1)}(z) \right\} \otimes \tilde{g}(x/z, t) \end{aligned} \quad (15)$$

where $\Delta \hat{P}_{qq}^{(1)}(z)$ and $\hat{P}_{Sg}^{(1)}(z)$ have the form

$$\begin{aligned} \Delta \hat{P}_{qq}^{(1)}(z) &= C_F N_F F_{qq}(z) \\ \hat{P}_{Sg}^{(1)}(z) &= \frac{1}{2}C_A N_F F_{qg}^{(1)}(z) + \frac{1}{2}C_F N_F F_{qg}^{(2)}(z) \end{aligned} \quad (16)$$

and the relevant expressions for the F 's can be read off from [2]. As in the non-singlet case, we can combine (12,15) and the evolution of the gluon into a 'one-step' evolution of the singlet structure function [7] correct to $\mathcal{O}(\alpha_S^2)$ which is of the form

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} F_S(x, t) \\ \tilde{g}(x, t) \end{pmatrix} &= \left\{ \left(\frac{\alpha_S}{2\pi}\right) \mathcal{P}_S^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \right. \\ &\times \left. \left\{ \mathcal{P}_S^{(1)}(z) - \frac{1}{4}\beta_0 \mathcal{D}^{(1)}(z) + \mathcal{E}^{(1)}(z) \right\} \right\} \\ &\otimes \begin{pmatrix} F_S(x/z, t) \\ \tilde{g}(x/z, t) \end{pmatrix} \end{aligned} \quad (17)$$

where $\mathcal{P}_S^{(0)}$, $\mathcal{P}_S^{(1)}$, $\mathcal{D}^{(1)}$ and $\mathcal{E}^{(1)}$ are given in [7].

Using (8) we have again that the effect of changing the scale $Q^2 \rightarrow Q^2(1-z)/z$ is to generate, at $\mathcal{O}(\alpha_S^2)$, the logarithm terms which explicitly appear in both the $\overline{\text{MS}}$ gluon and quark coefficient functions. So in addition to (10) we take

$$B_g(z) \longrightarrow 2N_F[8z(1-z) - 1] \quad (18)$$

For the gluon evolution we simply insert counter terms involving $\phi(z)$ to restore (17) at $\mathcal{O}(\alpha_S^2(t))$ generated by the scale change.

$$\frac{d}{dt} \tilde{g}(x, t) = \left\{ \frac{1}{z} \left[\left(\frac{\alpha_S}{2\pi}\right) z \hat{P}_{gg}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 \right. \right.$$

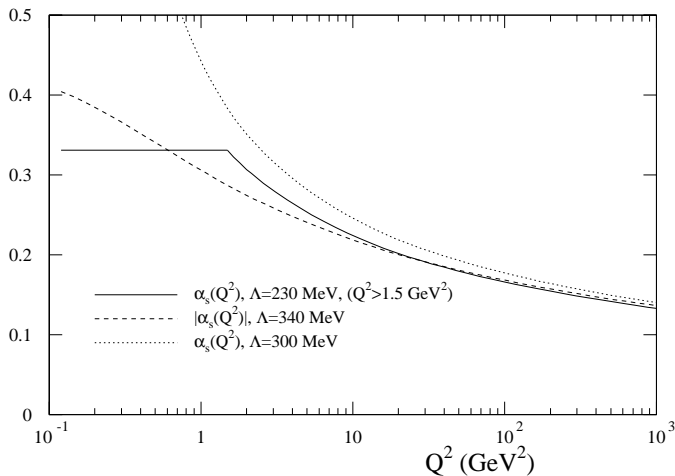


Fig. 1. Running couplings used in the various fits. The solid line shows $\alpha_S(|\tilde{W}^2|)$ versus \tilde{W}^2 , the dashed line shows $|\alpha_S(\tilde{W}^2)|$ versus \tilde{W}^2 , the values of $\Lambda_{\overline{\text{MS}}}$ being indicated. The dotted line shows, for the sake of comparison versus $\alpha_S(Q^2)$ versus Q^2 , the running coupling used in the standard MRST fit

$$\begin{aligned}
 & \times z \left(\hat{P}_{gg}^{(1)}(z) + \frac{1}{2} \beta_0 \hat{P}_{gg}^{(0)}(z) \ln[\phi(z)] \right) \Big|_+ \\
 & - \delta(1-z) \int_0^1 dz \left[\left(\frac{\alpha_S}{2\pi} \right) \hat{P}_{Sg}^{(0)}(z) \right. \\
 & \left. + \left(\frac{\alpha_S}{2\pi} \right)^2 \hat{P}_{Sg}^{(1)}(z) \right] \otimes \tilde{g}(x/z, t) \\
 & + \left\{ \left(\frac{\alpha_S}{2\pi} \right) \hat{P}_{gS}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi} \right)^2 \left(\hat{P}_{gS}^{(1)}(z) \right. \right. \\
 & \left. \left. + \frac{1}{2} \beta_0 \hat{P}_{gS}^{(0)}(z) \ln[\phi(z)] \right) \right\} \otimes \tilde{q}_S(x/z, t) \quad (19)
 \end{aligned}$$

While in the non-singlet case a single change of scale for α_S ((11)) can be chosen to absorb all the N_f dependence at $\mathcal{O}(\alpha_S^2)$, the situation is more complicated here and a similar BLM procedure would involve different choices of scale for the quark singlet and gluon evolution. An alternative to (17) would be a one step evolution expressed in terms of physical structure functions only, $F_S(x, t)$ and $F_L(x, t)$ in which case the modifications due to $Q^2 \rightarrow \phi(z)Q^2$ could be entirely absorbed at $\mathcal{O}(\alpha_S^2)$ into the coefficient functions $B_q(z)$ and $B_L(z)$.

3 Fitting the DIS data

A practical problem that arises in using $\phi(z) = (1-z)/z$ is simply that the integrations involved in the convolution integrals run up to $z = 1$ and so we must adopt some sensible approach for computing the running coupling at low values of the scale. Since the argument \tilde{W}^2 of α_S is actually timelike, then beyond leading order the coupling is complex. (Recall that the perturbation expansion is derived strictly in spacelike region and then continued analytically to the timelike region.) We consider two possibilities (i) compute $\alpha_S(|\tilde{W}^2|)$ in terms of $\Lambda_{\overline{\text{MS}}}$ in the standard

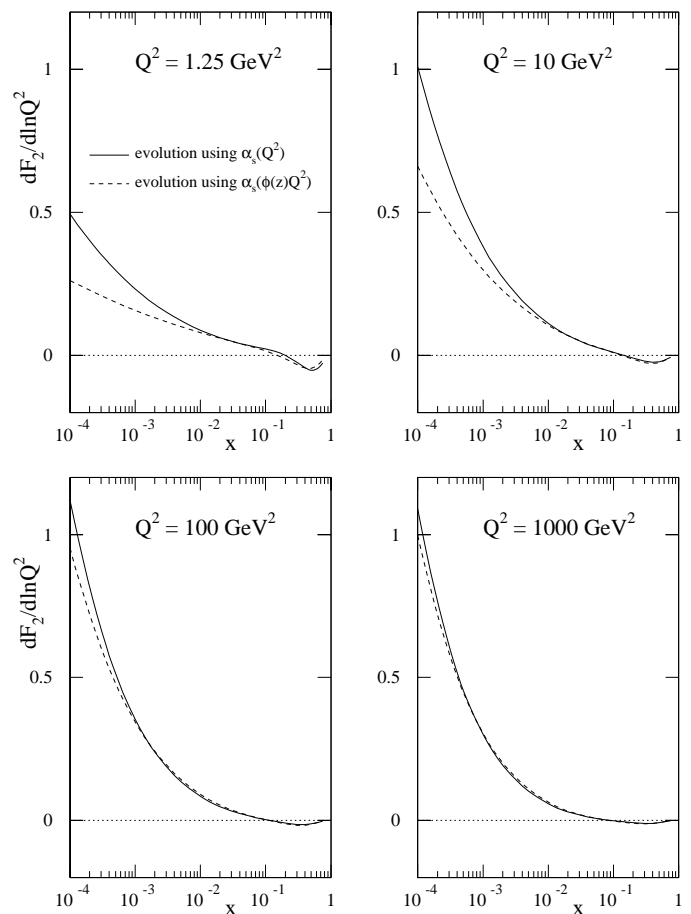


Fig. 2. Comparison of the evolutions with scales \tilde{W}^2 and Q^2 from a common set of parton distributions at $Q_0^2 = 0.5 \text{ GeV}^2$ and with a common value of $\Lambda_{\overline{\text{MS}}}$

NLO way but ‘freeze’ its value for $|\tilde{W}^2| < Q_f^2$ and (ii) use $|\alpha_S(\tilde{W}^2)|$ as the running coupling. This latter variable is claimed [8] to be a far more efficient expansion parameter (since it resums large terms involving π^2 arising from the analytic continuation) and grows only weakly as the argument becomes very small. The imaginary part of α_S is computed from the modulus at NLO,

$$\text{Im}(\alpha_S) = \frac{\beta_0}{4} |\alpha_S|^2 + \frac{\beta_1}{16\pi} |\alpha_S|^3 \quad (20)$$

Figure 1 displays the running couplings used in our fits compared with the α_S used by a conventional fit using a z -independent scale, the values of $\Lambda_{\overline{\text{MS}}}$ giving the best fit to the DIS data being shown. For the choice $\alpha_S(|\tilde{W}^2|)$ the quality of the fit is not overly sensitive to the precise value of the scale at which the coupling is frozen and we take a value around 1 GeV^2 .

As with the usual analysis of DIS structure functions, the parton distribution functions (pdf’s) are parametrised at some starting scale, Q_0^2 , but now evolved according to (7,15,19). In the recent MRST fits [9], the starting scale is $Q_0^2 = 1 \text{ GeV}^2$ and the gluon at small x is suppressed (so-called ‘valence-like’ gluon) at this Q^2 in order to de-

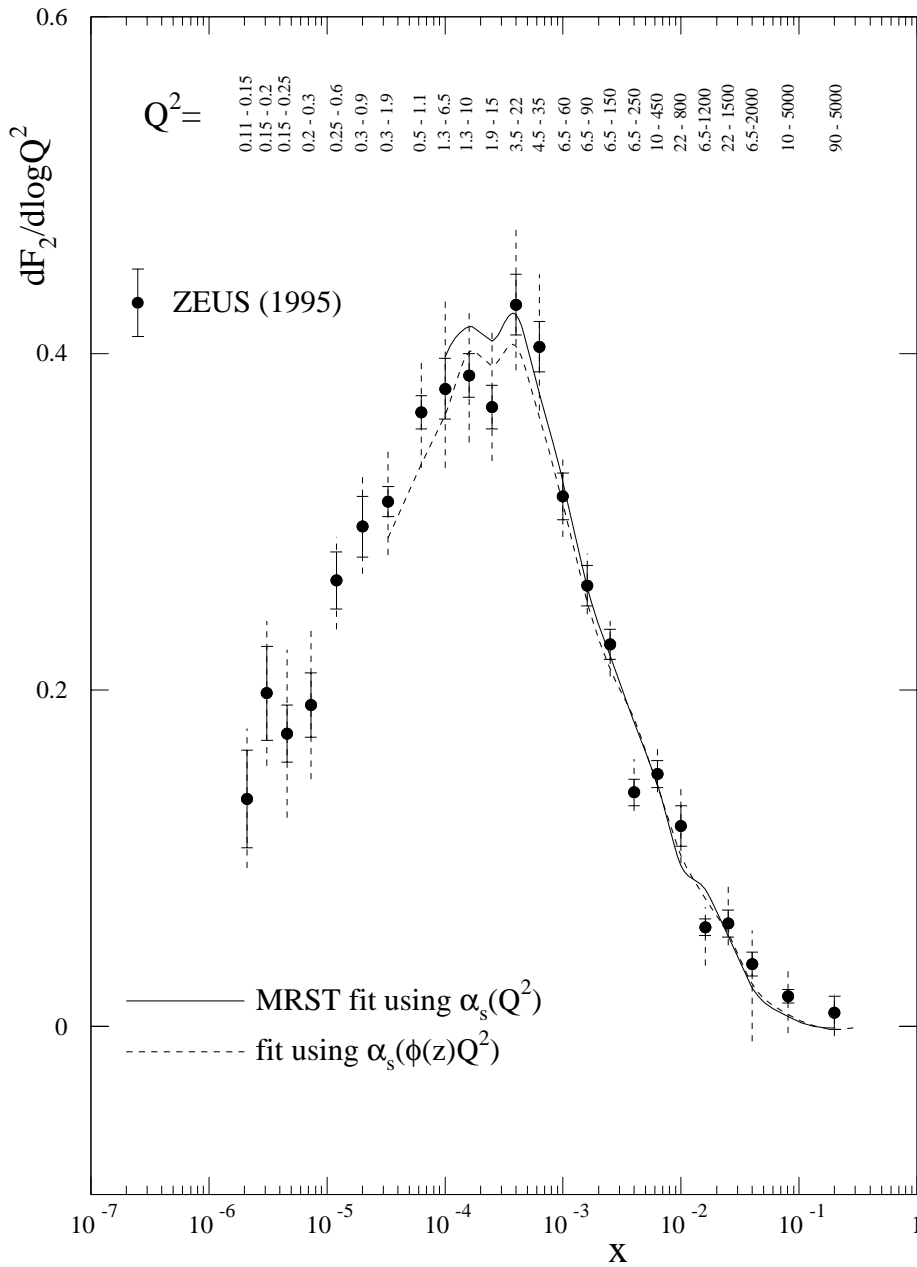


Fig. 3. Slope of F_2 for different x values measured by ZEUS[12]. Note the strong correlation between the value of x and the mean value of Q^2 . The data are compared to fits to DIS data using a scale Q^2 or a scale \tilde{W}^2

scribe the relatively small slope $dF_2/d\ln Q^2$ of the structure function observed at HERA [10,11] at low x and low Q^2 . At low x , the mean value of z in the evolution is also low implying that the scale \tilde{W}^2 is much larger than Q^2 and hence the gluon evolution is naturally suppressed at low x . In Fig. 2 we show a comparison of the structure function evolution for a common set of pdf's at Q_0^2 according to the two scales. The starting scale is $Q_0^2 = 0.5$ GeV² and it is clear that evolving with the scale \tilde{W}^2 leads to dramatically slower evolution at low x and low Q^2 , even with a common value of $\Lambda_{\overline{\text{MS}}}$.

The concept of a gluon distribution vanishing as $x \rightarrow 0$ (even at low Q^2) seems unnatural and leads to problems when attempting to evolve down in Q^2 since physical quantities, such as F_L quickly become negative. The

comparison in Fig. 2 shows we can instead start with a larger gluon and in fact we find the data can be fitted a gluon distribution which is actually singular for Q_0^2 as low as 0.5 GeV².

DIS data fitted include the HERA data [10–12], BCDMS [13], E665 [14], SLAC [15] and NMC [16]. We label the two types of fits as type (i) or type (ii) depending on the choice $\alpha_S(|\tilde{W}^2|)$ or $|\alpha_S(\tilde{W}^2)|$. There has been much interest in the values of the slope $dF_2/d\ln Q^2$ shown by the low x , low Q^2 1995 ZEUS data [12] that has prompted speculation about a possible breakdown of the standard theory [17]. A type (i) fit gives a satisfactory description of the slopes observed by ZEUS without the need to invoke a ‘valence-like’ gluon distribution at low Q^2 – as shown in Fig. 3 and as anticipated above. Because of the more

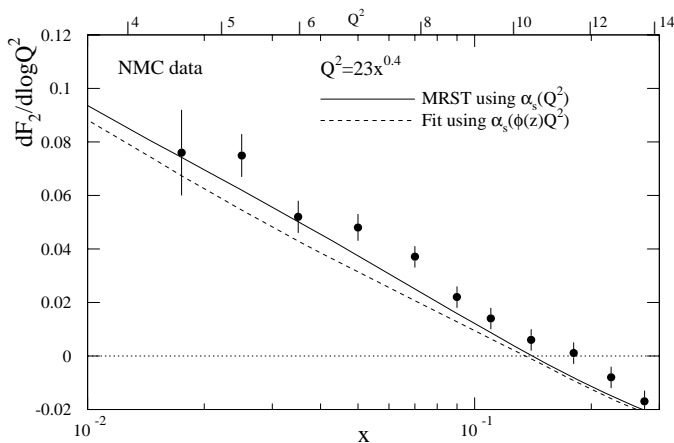


Fig. 4. Slope of F_2 for different x values measured by NMC[16]. Again the values of x and the mean value of Q^2 are correlated. The data are compared to fits to DIS data using a scale Q^2 or a scale \bar{W}^2

conventional behaviour of the gluon at small x one can evolve to values of Q^2 below 1 GeV². At the starting scale $Q_0^2 = 0.5$ GeV² the gluon has the form

$$xg(x, Q_0^2) = 1.25x^{-0.046}(1-x)^{5.52}(1 + 0.032\sqrt{x} + 5.66x) \quad (21)$$

which is virtually ‘flat’ at small x .

Fig. 3 clearly shows a good description of the F_2 slopes in the HERA range both for the conventional type of fit and for the new type (i) fit using a z -dependent scale. Comparing in detail the quality of the fits, we obtain

	H1	ZEUS	BCDMS	NMC	SLAC	E665
No. data pts	221	216	174	130	70	53
χ^2 (MRST)	164	264	249	143	116	56
χ^2 (type (i) fit)	164	270	243	170	134	52
χ^2 (type (ii) fit)	178	298	260	223	156	48

In the new fits, the cuts applied are the same as for MRST, in particular $Q^2 > 2$ GeV² except for the HERA data where we take $Q^2 > 1.5$ GeV². For the HERA data with $Q^2 > 1.5$ GeV², fit (i) achieves virtually the same quality as the conventional type fit but can, in addition, give a good description down to $Q^2 = 1$ GeV². The type (ii) is slightly worse for the HERA data. For the ‘intermediate’ x region covered by NMC, the new fits are not as successful as the conventional MRST fit, especially the type (ii) fit, and this is due to problem of trying to describe the Q^2 slopes of the NMC data while simultaneously being consistent with the slopes measured at HERA. The MRST fit already underestimates the slopes observed by NMC and the new fits only serve to emphasise the disagreement as shown in Fig. 4. Type (ii) fit being even worse indicates that the results are sensitive to the precise nature of the coupling at very low values of the scale.

4 Conclusions

Although the motivation for evolving partons with a scale $\phi(z)Q^2$ where $\phi(z) = (1-z)/z$ stems from a procedure for resumming terms which are potentially large as $x \rightarrow 1$, it is worth exploring the phenomenological consequences for quarks and gluons over the whole x region. In fact we did examine whether the large x region was better described in terms of a running coupling with the choice of scale $Q^2(1-z)/z$ - the hope being that the observed strong Q^2 observed at values of x beyond 0.6 might be absorbed by such a choice instead of conventional higher twist contribution (see [19] for example) but we found no evidence for this. The change of scale $Q^2 \rightarrow \phi(z)Q^2$ generates corrections at $\mathcal{O}(\alpha_s^2)$ which already appear explicitly as the logarithm terms in the $\overline{\text{MS}}$ coefficient functions, so evolving with the z -dependent scale implies somewhat simpler expressions for the coefficient functions. It should be remembered that our change of scale is a change of renormalisation scheme and in order to maintain the expressions for the evolution of the physical F_2 structure function to $\mathcal{O}(\alpha_s^2)$ we must modify the coefficient functions and the relevant splitting functions as shown in Sects. 1 and 2. Also the regularising counter terms (proportional to $\delta(1-z)$) are more complicated due to α_s depending on z .

Performing fits with two alternative prescriptions for handling the running coupling at low values of the scale leads to a preference for the simpler choice of freezing its value at around 1 GeV². In this case one can get acceptable fits to the DIS data but no improvement over the standard procedure is observed; in fact the problem of trying to reconcile the slopes $dF_2/d\ln Q^2$ measured by NMC with those measured at HERA is actually aggravated. This subtle Q^2 dependence of the slopes has only been successfully described when effects beyond standard DGLAP physics are included. In particular Thorne [18] was able to achieve a high degree of consistency between all the DIS data sets with a leading order, renormalisation scheme consistent calculation which included leading $\ln(1/x)$ terms¹.

An advantage which has been gained in using the scale $Q^2(1-z)/z$ is that the gluon distribution at low Q^2 seems more ‘natural’ than the ‘valence-like’ gluon which the standard DGLAP description finds necessary to account for the small values of the slopes observed by ZEUS [12]. We therefore expect that quantities dominantly governed by the gluon are sensitive to the change of scale, in particular the longitudinal structure function at low x , low Q^2 is quite different. Fig. 5 shows a comparison between predictions for F_L at $Q^2 = 1$ GeV² resulting from fits to F_2 using Q^2 or \bar{W}^2 as the choice of scale. The curious ‘dip’ of the MRST curve is due to the suppressed gluon contribution at low x while the smoother behaviour of the fit with the z -dependent scale reflects a dominance of the gluon contribution at small x .

¹ In an analysis of NLO corrections to BFKL equation, Thorne [20] has shown how an x -dependent choice of scale can effectively describe corrections at low x

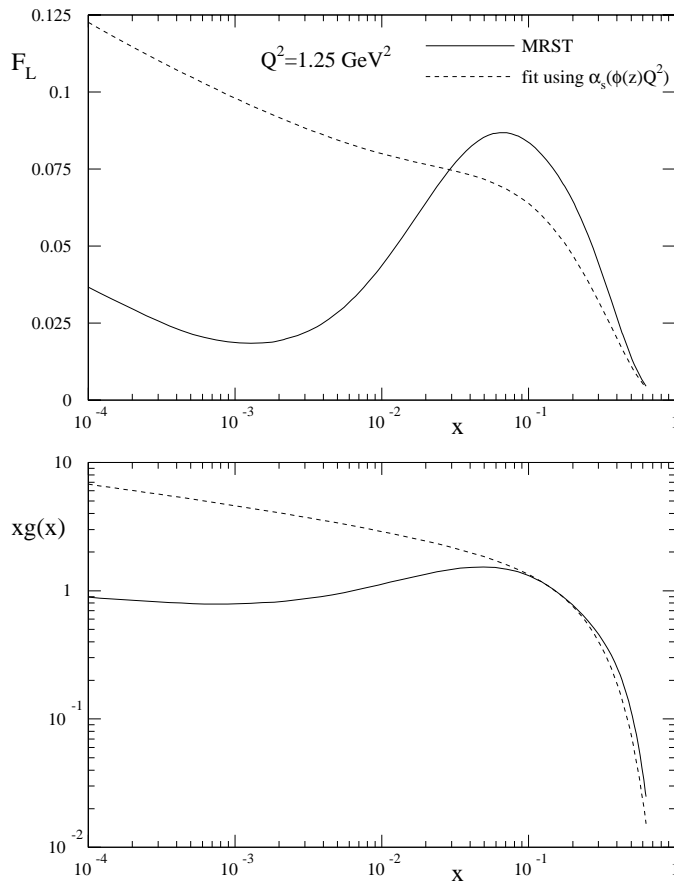


Fig. 5. A comparison of $F_L(x, Q^2 = 1.25)$ predicted by two fits, one (MRST) using the standard evolution scale, the other the type (i) fit using a scale \tilde{W}^2 . Also shown are the respective gluon distributions at the same $Q^2 = 1.25 \text{ GeV}^2$ which are responsible for driving F_L

In summary, the effects of resumming some of the $\log(1-x)$ terms through the change of scale does not lead to any improvement phenomenologically though it is consistent with a significantly different, and perhaps rather more natural, form of the gluon distribution at very low values of Q^2 . The exercise indicates that there is therefore a degree of uncertainty in the detailed nature of the gluon density at such low scales.

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